# Using a Pendulum to Calculate Earth's Gravitational Constant 

(g)

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#### Abstract

Using a pendulum to calculate Earth's gravitational constant, varying the length of the pendulum would not cause any change in the constant found. To test this, a string's length was being changed and the gravitational constant was calculated using physics equations to see if there is a correlation between the gravitational constant and the string length. It was found that no correlation existed between them.

\section*{Introduction}

The purpose of the experiment was to calculate Earth's gravitational constant, $g$, using a pendulum. Specifically, the pendulum's period of oscillation will define $g$ using the equation $g=4 \pi^{2}\left(L / T^{2}\right)$ where $L$ is the length of the pendulum and $T$ is the period of oscillation [1]. This equation can be found using Isaac Newton's second law of motion. Newton's second law states that the acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object [2]. Simply put, $\mathrm{F}=\mathrm{ma}$ where F is the net force, $m$ is the mass of an object, and a is the acceleration of the object.


Figure 1: Diagram of pendulum with force components shown.


For a pendulum, the net force is $F=-m g \theta$ where $m$ is the mass of the object used in the pendulum, g is the acceleration due to gravity which is what will be calculated, and $\theta$ is the angle created from the pendulum vertically. In radians, the arc length in a circle is related to its radius ( L in this instance) by $\mathrm{s}=\mathrm{L} \theta$, so that $\theta=\mathrm{s} / \mathrm{L}$ [3]. Now, the expression for the net force is $F=-m g s / L$ which is of the form $F=-k x$, the force for objects in oscillation. In the equation, $F=-k x$, the constant $k$ is equal to $-\mathrm{mg} / \mathrm{L} . \mathrm{k}$ will be used in the period of the pendulum which is given by the equation $T=2 \pi \sqrt{ }(\mathrm{mk})$ where $T$ is the period. After substituting $k$ into the equation for the period of a pendulum, $T=2 \pi \sqrt{ }\{\mathrm{~m} /(\mathrm{mgL})\}$ or $T=2 \pi \sqrt{(L / g)}[3]$. Solving for acceleration due to gravity, $g=4 \pi^{2}\left(L / T^{2}\right)$.

It was hypothesized that as the length of the pendulum, L , was decreased, the value found for Earth's gravitational constant, $g$, will neither decrease nor increase.

## Objective

Calculating Earth's gravitational constant from the period of oscillation from a pendulum. Once measured, it will be discussed whether the length of the pendulum affects the gravitational constant found.

## Materials

- Meter-long string
- Washer bolt (regardless of mass)
- Stopwatch
- Scissors


## Procedure

1. Attach the washer to the string.
2. Hold the string so that it is completely vertical.
3. Hold the washer so that it creates an angle with the vertical.
a. The angle should be small, around 10 or less[3].
4. Release the washer and allow the pendulum to swing back and forth for 10 oscillations.
5. Record the time it took to complete the 10 oscillations.
6. Calculate $g$ using the equation $g=4 \pi^{2}\left(\mathrm{~L} / \mathrm{T}^{2}\right)$.
a. To calculate T, divide how long it took to complete the 10 oscillations by 10 .
7. Record data using the table below.

| Length of String, L, | Period, T , | $g$ |
| :---: | :---: | :---: |


| $(\mathrm{m})$ | $(\mathrm{s})$ | $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| :---: | :---: | :---: |
| 1.0 |  |  |
| 0.9 |  |  |
| 0.8 |  |  |
| 0.7 |  |  |
| 0.6 |  |  |

8. Repeat Steps $1-7$ by cutting the string by 10 cm increments for 5 trials.

Figure 2: Appropriate setup for experiment.


## Results

With the string length as 1 meter, the average time it took to complete one oscillation was 2.03 seconds. After substituting the known variables into the equation, $g=4 \pi^{2}\left(L / T^{2}\right), g$ was found to be 9.58 meters per second squared. When the string was cut and reduced to .9 meters or 90 cm , the average time to complete one oscillation was 2.01 seconds. Thus, g was calculated to be 8.79 meters per second squared. When the string was 0.8 meters or 80 cm , the period of oscillation was 1.80 seconds. With this length and period, $g$ was measured to be 9.74 meters per second squared. For a length of 0.7 meters or 70 cm , the period was 1.65 seconds and g was evaluated as 10.15 meters per second squared. When the length of the string was 0.6 meters or 60 cm , the period was 1.49 seconds and g was computed to be 10.67 meters per second squared. These results are recorded in Table 1 below.

Table 1: The period of one oscillation found (T) and the acceleration due to gravity calculated $(g)$ from varying string lengths (L).

| Length of String, L, <br> $(\mathrm{m})$ | Period, T, <br> $(\mathrm{s})$ | $g$ <br> $\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |
| :---: | :---: | :---: |
| 1.0 | 2.03 | 9.58 |
| 0.9 | 2.01 | 8.79 |
| 0.8 | 1.80 | 9.74 |
| 0.7 | 1.65 | 10.15 |
| 0.6 | 1.55 | 9.86 |

## Analysis

The data collected from the experiment supports the supposition that changing the length of the pendulum does not correlate with the gravitational constant calculated. This can be seen, for example, when the length of the string went from 1 m to 0.9 m . In this case, g decreases and went from $9.58 \mathrm{~m} / \mathrm{s}^{2}$ to $8.79 \mathrm{~m} / \mathrm{s}^{2}$. However, when the length of the string went from 0.9 m to 0.8 m , g increased and went from $8.79 \mathrm{~m} / \mathrm{s}^{2}$ to $9.74 \mathrm{~m} / \mathrm{s}^{2}$. While there was no correlation between string length and acceleration due to gravity, there was a correlation found between string length and period where as the length decreased, the period would decrease as well. In this experiment, there are some potential errors that may have affected the results. The angle created from holding the washer was not constant and defined since it only had to be a small angle less than 10 degrees. Additionally, there could have been some air resistance throughout the experiment and possibly affected the oscillations. There are some ways to change this experiment so that the effects of such errors are lowered and further research can be conducted. If the angle created from holding the washer was measures and held constant, there would be less flaws in the experiment. On the other hand, for a future experiment, the length of the string can be held constant while the angle changes to see how the angle affects the gravitational constant. Instead of having one mass for the entire experiment, there can be multiple masses used with an extremely high difference between them to see how mass impacts tensions and consequently impacts the gravitational constant found. Additionally, instead of decreasing the string length, future experiments can have the length of the string increasing.

## Reference List

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